CONTINUOUS-TIME MODEL IDENTIFICATION USING REINITIALIZED PARTIAL MOMENTS - APPLICATION TO POWER AMPLIFIER MODELING

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Abstract: An application of continuous-time system identification to the modeling of a power amplifier used in mobile communication is proposed. Generally, the electronic components modeling uses discrete-time black-box models. In this work, from electronics engineer knowledge, a continuous-time transfer function model is chosen to represent amplifier transmittance. Hence, a parameter estimation is performed by the reinitialized partial moments method. This estimation method is used for the relatively insensitivity to the initial conditions and rough system a priori knowledge. The power amplifier continuous-time model allows to the electronics engineer an immediate physical interpretation.

Keywords: Continuous-time model identification, Reinitialized partial moments, Power amplifier, Modeling, Application to mobile communication.

1. INTRODUCTION

This paper deals with continuous-time model identification of dynamical systems from measured data and application to power amplifier modeling used in mobile communication.
During last two decades, there has been a new interest in continuous-time approaches (Unbehauen and Rao, 1998). An overview of approaches is given by (Unbehauen and Rao, 1987) (Young, 1981). In the recent paper (Garnier *et al.*, 2003*b*), continuous-time model identification tools included in the CONTSID Matlab toolbox (Garnier *et al.*, 2003*a*) are described and evaluated. Moreover, these tools are divided in three classes of methods: linear filters, integral methods and modulating functions. The Reinitialized Partial Moments (RPM) method presented and used in the present paper is included in integral methods class. The main idea of this class is to avoid the input-output time-derivatives calculation by performing integrations. In this class, the particularity of the RPM method is to use a time-shifting window for the integration and to perform an output noise filtering.
In mobile communication systems, the consumption is a crucial problem. For radio frequency application, the key challenge is the design of electronic components which provide high linear-

ity and efficiency. This paper presents a modeling of a Power Amplifier (PA) where the purpose is to improve the comprehension of amplifier transmittance effects. More particularly, the PA linear part is modeled by a continuous-time transfer function which will included in nonlinear block-oriented model. Generally, the electronic components modeling uses discrete-time black-box models. A continuous-time model in PA modeling does not seem to have been used previously in the literature on electronic area. The electronics engineer can be interpreted immediately the model in physical terms.

2. PRELIMINARIES

2.1 Time moment and partial moment

Let $f(t)$ be a function defined on $[0, \infty[$ with Laplace transform $F(s)$. The well-known n^{th} order time moment is defined by

$$M_n^f = \int_0^\infty t^n f(t)dt \tag{1}$$

$F(s)$ admits a Taylor series expansion about $s = 0$ given by

$$F(s) = \sum_{n=0}^{\infty} (-1)^n \frac{M_n^f}{n!} s^n \tag{2}$$

Let us define the partial time moment as a truncation on $[0, T[$ of M_n^f

$$M_n^f(T) = \int_0^T t^n f(t)dt \tag{3}$$

Notice that partial moments are also called truncated moments in nuclear physics among others.

2.2 Output model with partial moments

Consider a stable first order system given by the following differential equation

$$\dot{y}(t) = -a_0 y(t) + b_0 u(t) \tag{4}$$

Calculate the integral of $t\dot{y}(t)$ on $[0, T[$ to obtain the following output model

$$\begin{aligned} \hat{y}(T) &= -\hat{a}_0 \frac{\int_0^T ty(t)dt}{T} + \hat{b}_0 \frac{\int_0^T tu(t)dt}{T} + \frac{\int_0^T y(t)dt}{T} \\ &= -\hat{a}_0 \frac{M_1^y(T)}{T} + \hat{b}_0 \frac{M_1^u(T)}{T} + \frac{M_0^y(T)}{T} \end{aligned} \tag{5}$$

The output $\hat{y}(T)$ can be rewritten

$$\hat{y}(T) = \hat{a}_0 \alpha_0^y(T) + \hat{b}_0 \beta_0^u(T) + \gamma^y(T) \tag{6}$$

where $\alpha_0^y(T)$, $\gamma^y(T)$ and $\beta_0^u(T)$ are functions of partial moments and can be expressed in the following form

$$\begin{aligned} \alpha_0^y(T) &= -\int_0^T f_0(t)y(t)dt, \quad f_0(t) = \tfrac{t}{T} \\ \gamma^y(T) &= -\int_0^T f_1(t)y(t)dt, \quad f_1(t) = -\tfrac{1}{T} \\ \beta_0^u(T) &= \int_0^T f_0(t)u(t)dt \end{aligned} \tag{7}$$

In the equations (7), a input-output data filtering notion appears.
To simplify the presentation of this approach, in these preliminaries, we are considered only a simple first order system. An extension to a nth order system is given in (Trigeassou, 1987).

2.3 Properties of partial moments model

Suppose, in stochastic case, that the measured output is given by

$$y(t) = y_0(t) + v(t) \tag{8}$$

where $y_0(t)$ is free-noise output and $v(t)$ is a zero mean white noise with a variance σ_v^2.
Consider the estimation error $\varepsilon(T)$ that represents the difference between output (6) in deterministic and stochastic cases. Then, we obtain

$$\varepsilon(T) = \hat{a}_0 \alpha_0^v(T) + \gamma^v(T) \tag{9}$$

If we consider the sampled measured input-output data with sampling time t_s and $T = Kt_s$, the estimation error has a zero mean and a following variance (Trigeassou, 1987)

$$\begin{aligned} \sigma_e^2 = \sigma_v^2 \Big[& K \tfrac{(a_0 t_s)^2}{3} - a_0 t_s (1 + \tfrac{a_0 t_s}{2}) + \\ & \tfrac{1}{K} \left(1 + a_0 t_s + \tfrac{(a_0 t_s)^2}{6} \right) \Big] \end{aligned} \tag{10}$$

obtained by using simple square rule for integral calculus.
Now, it is easy to show (Trigeassou, 1987) that the estimation error variance is minimized if an optimum interval $T = T_{opt} = K_{opt} t_s$ is chosen where K_{opt} is near to the integer part of

$$\sqrt{3} \frac{\tau}{t_s} \tag{11}$$

τ is the time constant of the first order system and t_s respects the condition $t_s << \tau$.
In case of the estimation with variance minimum, the output model with partial moments has the fundamental output noise filtering property.

3. REINITIALIZED PARTIAL MOMENTS

3.1 Definition

The output model given by (6) has two properties

- linearity in parameters,
- output noise filtering.

However, this filtering occurs only for $T = T_{opt}$. For $T >> T_{opt}$, there is not filtering and an error accumulation causes the divergence of the variance of estimator $\hat{y}(T)$.
Therefore, to use the filtering property, it is necessary to work at the minimum variance and at each instant. It is possible by reinitializing partial moments for each time t. Thus we define the Reinitialized Partial Moment (RPM)

$$\hat{M}_n^f(t) = \int_0^{\hat{t}} \tau^n f(t - \hat{t} + \tau) d\tau \tag{12}$$

with $\hat{t} = K_{opt} t_s$ is called reinitialization time.

3.2 RPM Output model

(Trigeassou, 1987) demonstrates an extension of the output model (6) to the nth order and to the reinitialized partial moments.
Let us consider a stable SISO continuous-time system described by the strictly proper transfer function

$$H(s) = \frac{B(s)}{A(s)} \tag{13}$$

with

$$\begin{array}{l} B(s) = b_0 + \ldots + b_m s^m \ , \ m < n \\ A(s) = a_0 + \ldots + a_{n-1} s^{n-1} + s^n \end{array} \tag{14}$$

The output model with reinitialized partial moments is given by

$$\hat{y}(t) = \sum_{i=0}^{n-1} \hat{a}_i \alpha_i^y(t) + \sum_{j=0}^{m} \hat{b}_j \beta_j^u(t) + \gamma^y(t) \tag{15}$$

where

$$\begin{array}{l} \alpha_0^y(t) = -m(t) * y(t) \\ \beta_0^u(t) = m(t) * u(t) \\ \alpha_i^y(t) = -\frac{d^i m(t)}{dt^i} * y(t) \text{ for } 1 \leq i < n \\ \beta_j^u(t) = -\frac{d^j m(t)}{dt^j} * u(t) \text{ for } 1 \leq j \leq m \\ \gamma^y(t) = \left(\delta(t) - \frac{d^n m(t)}{dt^n}\right) * y(t) \\ \delta(t) \text{ is Dirac function} \\ m(t) = \frac{(\hat{t}-t)^n t^{n-1}}{(n-1)! \hat{t}^n} \text{ with } t \in \left[0, \hat{t}\right] \\ \text{and } * \text{ is a convolution product.} \end{array} \tag{16}$$

In the RPM output model (15), $\alpha_i^y(t)$, $\beta_j^u(t)$ and $\gamma^y(t)$ are functions of the reinitialized partial moments.

3.3 Discussion

In continuous-time system identification, a classical problem is to obtain the time-derivatives of the input-output signals. A large class of approaches based on filtering exists (Young, 1981)(Unbehauen and Rao, 1987)(Unbehauen and Rao, 1998). It is clear that the RPM output model (15) belongs to this class. The used filter $m(t)$ is a Finite Impulse Response (FIR) filter. Generally in the filtering approaches, the filter choice is the crucial step. For RPM output model, the only parameter to choose is the reinitialization time $\hat{t}$. Comparative studies (Dréano, 1993)(Jemni and Trigeassou, 1996)(Mensler *et al.*, 2000)(Garnier *et al.*, 2003*b*) prove the good performances of the RPM output model, and the relatively weak sensitivity of this approach to the filter choice and sample time.
In practice, the reinitialization time $\hat{t}$ is chosen to be twice the main time constant. For example, if the sampling time t_s is equal to the main time constant divided by 10, then we choose $\hat{t} = 20t_s$. Note that the RPM output model is not very sensitive to this choice, *i.e.* for the above example, both choices $\hat{t} = 15t_s$ and $\hat{t} = 25t_s$ give quite similar results. Moreover, for resonant systems, the reinitialization time can be chosen equal to twice the rising time of the step response.
Therefore a rough system knowledge about time constant, that can be obtained with a step response, allows to choose the RPM output model filter $m(t)$.
The transient effect of the filter $m(t)$ disappears after $\hat{t}$ since it is a FIR filter. On the other hand, we cannot use the first data for $t < \hat{t}$. A major advantage is that this technique is not sensitive to initial conditions.
The RPM output model is linear in parameters. Then the parameter estimation is given by Least-Squares (LS). Unfortunately, the estimate is biased as this is generally the case with least-squares. The Instrumental-Variable (IV) method with parallel model can be used to eliminate this bias.

3.4 Implementation

The RPM output model (15) can be rewritten as linear regression form

$$\hat{y}(t) = \varphi^T(t)\theta + \gamma^y(t) \tag{17}$$

where

$$\begin{aligned}\varphi(t) &= \left[\alpha_0^y(t), \ldots, \alpha_{n-1}^y(t), \beta_0^u(t), \ldots, \beta_m^u(t)\right]^T \\ \theta &= \left[\hat{a}_0, \ldots, \hat{a}_{n-1}, \hat{b}_0, \ldots, \hat{b}_m\right]^T\end{aligned} \tag{18}$$

Assume that we have measured N values of input and output, the least-squares estimate of θ is given by

$$\begin{aligned}\hat{\theta}^{LS} &= \left[\frac{1}{N-\hat{k}} \sum_{i=\hat{k}}^{N} \varphi(it_s)\varphi^T(it_s)\right]^{-1} \\ &\quad \frac{1}{N-\hat{k}} \sum_{i=\hat{k}}^{N} \varphi(it_s)\left(y(it_s) - \gamma^y(it_s)\right)\end{aligned} \tag{19}$$

where $\hat{k}$ corresponds to $\hat{t} = \hat{k}t_s$.

The Instrumental-Variable method gives the following estimate

$$\begin{aligned}\hat{\theta}^{IV} &= \left[\frac{1}{N-\hat{k}} \sum_{i=\hat{k}}^{N} \zeta(it_s)\varphi^T(it_s)\right]^{-1} \\ &\quad \frac{1}{N-\hat{k}} \sum_{i=\hat{k}}^{N} \zeta(it_s)\left(y(it_s) - \gamma^y(it_s)\right)\end{aligned} \tag{20}$$

where

$$\begin{aligned}\zeta(t) &= \left[\alpha_0^z(t), \ldots, \alpha_{n-1}^z(t), \right. \\ &\quad \left. \beta_0^u(t), \ldots, \beta_m^u(t)\right]^T\end{aligned} \tag{21}$$

and $z(t)$ is the response of the parallel model $H(s, \hat{\theta}) = \hat{B}(s)/\hat{A}(s)$.
The method can be used iteratively with a parallel model intialized with least-squares estimates.
In the RPM output model (15), $\alpha_i^y(t)$, $\beta_j^u(t)$ and $\gamma^y(t)$ are given by convolution products between filter impulse response $m(t)$ or its derivatives and input-output signals.
In practice, we use the following expression (Coirault, 1992)(Dréano, 1993)

$$\begin{aligned}\alpha_i^y(t) &= -\int_0^{\hat{t}} f_i(\mu)y(t - \hat{t} + \mu)d\mu \\ \beta_j^u(t) &= \int_0^{\hat{t}} f_j(\mu)u(t - \hat{t} + \mu)d\mu \\ \gamma^y(t) &= -\int_0^{\hat{t}} f_n(\mu)y(t - \hat{t} + \mu)d\mu\end{aligned} \tag{22}$$

with

$$\begin{aligned}f_0(\mu) &= \frac{\mu^n(\hat{t}-\mu)^{n-1}}{(n-1)!\hat{t}^n} \\ f_i(\mu) &= -\frac{df_{i-1}(\mu)}{d\mu}\end{aligned} \tag{23}$$

The following recursive form allows to compute $f_i(\mu)$

$$\begin{aligned}f_i(\mu) &= \frac{(-1)^i}{(n-1)!\hat{t}^n} \sum_{j=0}^{i} (-1)^j \frac{i!}{j!(i-j)!} \\ &\quad \frac{(n-1)!n!}{(n-j-1)!(n-i+j)!}\mu^{n-i+j}(\hat{t} - \mu)^{n-j-1} \\ &\quad \text{for } n - j - 1 \geq 0\end{aligned} \tag{24}$$

To compute the integrations in (22), we use the Simpson's rule, *e.g.* for $\alpha_i^y(t)$

$$\begin{aligned}\alpha_i^y(t) &= -\frac{t_s}{3} \sum_{k=2}^{\hat{k}} \left[f_i((k-2)t_s)\right. \\ &\quad y(t - (\hat{k} - l + 2)t_s) + \\ &\quad 4f_i((k-1)t_s)y(t - (\hat{k} - l + 1)t_s) + \\ &\quad \left. f_i(kt_s)y(t - (\hat{k} - l)t_s)\right] \\ &\quad \text{where } k \text{ and } \hat{k} \text{ are even}\end{aligned} \tag{25}$$

The function $\beta_i^u(t)$ can be computed like in (25). For particular case of piecewise constant input, *e.g.* the case of an input generated by digital to analog converter, we use the square rule for the integration. Then, the following expressions are used

$$\beta_j^u(t) = \sum_{k=0}^{\hat{k}-1} F_i^{sq}(kt_s)u(t - (\hat{k} - k)t_s) \tag{26}$$

with

$$\begin{aligned}F_i^{sq}(kt_s) &= \frac{(-1)^i}{(n-1)!(\hat{k}t_s)^n} \sum_{j=0}^{i} (-1)^j \\ &\quad \frac{i!}{j!(i-j)!} \frac{(n-1)!n!}{(n-j-1)!(n-i+j)!} \\ &\quad \sum_{r=0}^{n-j-1} (-1)^r \frac{(n-j-1)!}{r!(n-j-1-r)!}(\hat{k}t_s)^{n-j-1-r} \\ &\quad \left\{\frac{((k+1)t_s)^{n-i+j+r+1} - (kt_s)^{n-i+j+r+1}}{n-i+j+r+1}\right\}\end{aligned} \tag{27}$$

In the CONTSID Matlab toolbox (Garnier *et al.*, 2003*a*), the RPM output model estimation with Instrumental-Variable method (20) is included. CONTSID can be downloaded from *http://www.cran.uhp-nancy.fr/contsid/*, and the function *ivrpm* allows to obtain a RPM output model estimate.

4. APPLICATION TO POWER AMPLIFIER ESTIMATION

A power amplifier is a circuit for converting dc-input power into significant amount of Radio Frequency and Microwave output power. This component can consume a major fraction of the power used by the system and also distort the transmitted signal, introducing additional spectral components within the signal frequency band (Kenington, 2000). Thus, the key challenge amplifier design for mobile communication systems is the design of amplifiers, which provide high output power, linearity and efficiency. For that, the modeling step is indispensable to a good comprehension of amplifier transmittance effects.

In our application, the system is composed from the PA circuit associated with modulation and demodulation components. In this case, the used signals are modulating input and demodulating output.

4.1 Power Amplifier model

The main advantage of the continuous-time methods over the alternative and better known discrete methods is that they provide an input-output relations whose parameters can be interpreted immediately in physically terms. In the PA case, engineers and users can easily interpret estimation results such as a power gain conversion and a cut-off frequency.

In the conventional behavioral model, the characteristics of an amplifier are specified completely by its behavior with single tone input, including input voltage V_{in}, gain conversion G associated with amplitude to amplitude transfer (AM-AM characteristic) and output voltage V_{out}. The input to output relationship of the continuous PA model may be represented with a differential equation

$$\frac{d^n}{dt^n}V_{out} + \sum_{k=0}^{n-1} a_k \frac{d^k}{dt^k}V_{out} = \sum_{k=0}^{m} b_k \frac{d^k}{dt^k}V_{in} \quad (28)$$

The coefficients a_i and b_i are real scalars that define the model. Thus, the input-output relation can be expressed in Laplace domain with the continuous-time transfer function

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{\sum_{i=0}^{m} b_i \cdot s^i}{s^n + \sum_{i=0}^{n-1} a_i\, s^i} \quad (29)$$

4.2 PA experimental setup

In this section, we illustrate through experimentations, performance of the PA continuous-time model identification based on Reinitialized Partial Moments.

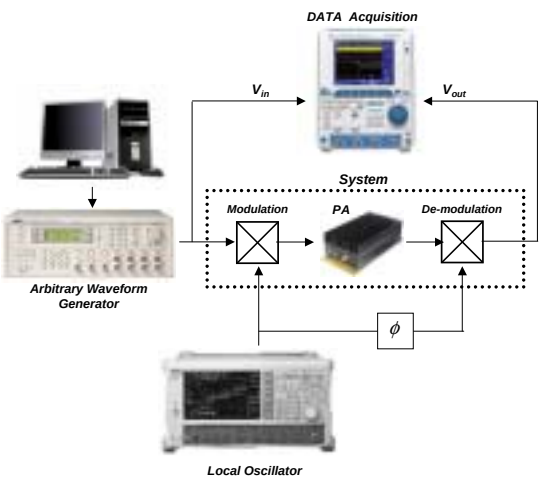

Fig. 1. PA Experimental setup

For the experimental investigation, a commercial 700-MHz/4,2-GHz MINI-CIRCUITS is used (ZHL-42 Model). Input and output data are obtained from YOKOGAWA Digital Oscilloscope.

Identification algorithm needs persistent excitation to provide appropriate estimation. Indeed, modulated signals are required to excite both steady-state (low frequency) and process dynamics (medium to high frequency). This excitation is performed with a Pseudo Random Binary Sequence (P.R.B.S) baseband pulse as shown in figure 2. To work in the PA linear area, the input amplitude is chosen equal to $0,12$ V.

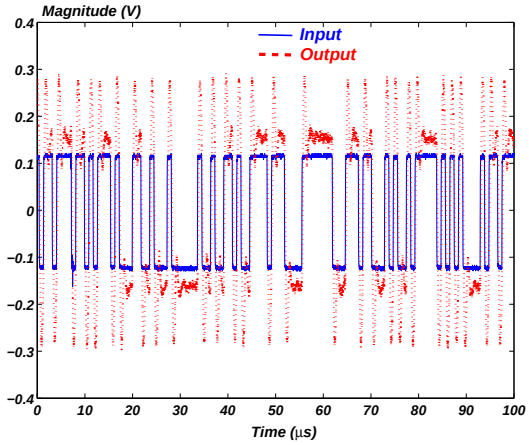

Fig. 2. Input/output data set

All modulation signals are delivered by a TTi 40 MHz Arbitrary Waveform Generator connected to PC control. The local oscillator frequency is 900 MHz obtained from 300-KHz/2,75-GHz Digital Modulation Signal Generator (ANRITSU MG 3660A). The data acquisition is done at a sampling period equal to 10 ns using anti-alias filters.

4.3 Tests results

For the estimation algorithm, the numerator and denominator orders of the transfer function have to be given. From the empirical transfer function estimate (Djamai *et al.*, 2005), it may be noticed that the system has at least one complex pole pair (2nd order resonant system) described by

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{b_1\, s + b_0}{s^2 + a_1\, s + a_0} \quad (30)$$

This structure is justified by the real dynamic of Radio Frequency Power Amplifier who have a resonant mode around 1 MHz and a high pass behavior introduced by modulation/demodulation elements. Notice that it is so easy to have a good approximation of model order using a continuous representation according to a discrete one. Thus, we define the estimated parameter vector

$$\underline{\theta} = [\, a_0 \;\; a_1 \;\; b_0 \;\; b_1 \,]^T$$

After running the IV method (20) with reinitialization time $\hat{t} = 0,2\,\mu s$, we obtain

$$\begin{cases} \hat{a}_0 = 3,38.10^{13} \\ \hat{a}_1 = 3,12.10^{6} \\ \hat{b}_0 = 4,48.10^{13} \\ \hat{b}_1 = 1,84.10^{6} \end{cases} \Rightarrow \begin{cases} G = 1,325 \\ f_c = 0,925 \text{ MHz} \end{cases}$$

where the PA gain G and the resonant frequency f_c are calculated using $\hat{a}_0$, $\hat{a}_1$, $\hat{b}_0$ and $\hat{b}_1$ values.

Figure 3 compares the simulated model output (dotted line) with the measured output for a different experiment (solid line). It may be seen that the simulated output follows the measured one.

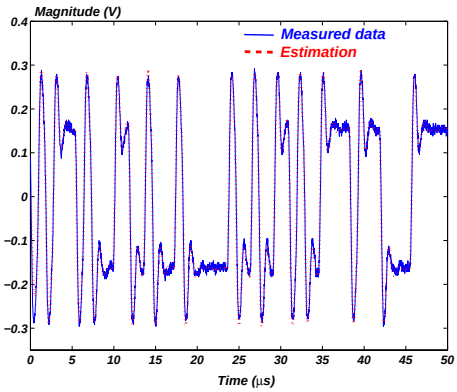

Fig. 3. Comparison of time domain measurement and estimation

As observed in figure 4, the identification residuals (estimation error) are negligible and don't exceed $0,04$ V.

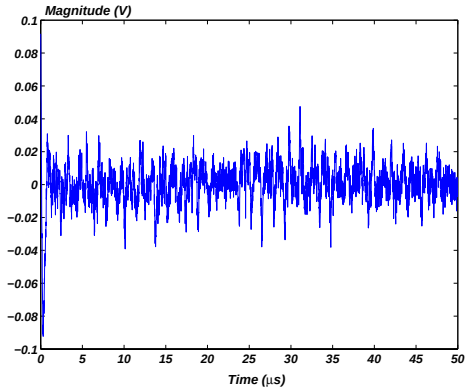

Fig. 4. Identification residuals

5. CONCLUSION

In this paper an application of continuous-time system identification to the modeling of a power amplifier is proposed. The use of a continuous-time model in radio frequency component area is original. Generally, discrete-time black-box models is used. The main advantage is the direct physical interpretation of the model by electronics engineers.

An RPM model is used to represent the continuous-time transfer function. The main advantages are that the RPM scheme is not sensitive to initial conditions, the filter needs not a precise system knowledge and an output noise filtering is made.

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