A Topological Approach for Automated Unstructured Meshing of Complex Reservoir

V. Gauthier (University of Poitiers / XLim), A. Arnould (University of Poitiers / XLim), H. Belhaouari (University of Poitiers / XLim), S. Horna (University of Poitiers / XLim), M. Perrin (Geosiris), M. Poudret (Geosiris) & J.F. Rainaud (Geosiris)

Abstract

The estimation of petroleum reserves entails complex fluid simulations mostly based on finite volume methods. These simulations are operated on 3D meshed reservoir models produced by a complex and poorly automated chain of operations. This paper proposes a mesh building methodology, which uses geological rules for building reservoir meshes in a semi-automated way.

We start from a surface structural model and a description of the stratigraphy bundled together thanks to the industry standard RESQML. We construct a structural framework based on generalized map topological structures. These structures include topological boundary relations between the represented geological objects (horizons, faults, units) and some dedicated data attached to the topological cells (vertices, faces, volumes, etc.), such as geometry or geological labels (e.g. names, relative ages, deposit methods). In particular, on a single topological representation, we can attach two different geometric representations that respectively describe the present-day layer geometry and the original positions of the various layers in their “deposition space”.

Using a dedicated rule-based language, we introduce a set of topological and geometric operations based on the geological interpretation that allow an automated building of the structural framework, on which the reservoir meshes will be implemented. This language allows a fast prototyping of complex operations (boolean operations for instance) and it guarantees the geological and topological model consistency.

From this fully consistent structural framework, we can automatically create various conformal unstructured 3D meshes organized in layers. These meshes agree both with the topology induced by the succession of deposition, erosion and tectonic events that constitute the local geological history, and with the peculiarities of the used fluid flow simulators. A use-case is presented to demonstrate the feasibility of our method.

Introduction

A significant part of a geological reservoir study depends on fluid flow simulations operated on 3D geological models (sets of horizons and faults). The construction of these models is based on interpretations formulated by geologists using the data collected by seismic and drilling surveys. Fluid flow simulations operated on these models provide additional subsurface information about the economic potential or the storage capacity of the studied reservoir. These are essential in the case of hydrocarbon reservoirs for estimating the volume of petroleum that can be extracted.

Resting on material balance equations [IMMA10], fluid flow simulations for petroleum exploration take into account both the physical properties of the rocks (porosity, permeability, density) and those of the fluids that they contain (porosity, permeability, density, viscosity, pressure). The efficiency of the involved computational methods (finite elements or finite differences) greatly depends on the quality of the used 3D meshes. Meshing the geological units that constitute a reservoir and its surrounding is therefore a difficult problem since meshes need not only to meet computation requirements but also to handle structurally complex geological environments. In particular, the stratigraphic units that constitute the reservoir environment are not always flat. They may be more or less deformed by folds and interrupted by faults whose orientations have little to do with the orientations of the layers themselves. These various requirements are contradictory and make 3D geological meshing an actual challenge for the geoscientists and mathematicians involved in earth modeling.

In the present paper we propose a new method to face this challenge. In order to ease the meshing process, our method intends to take better account of the evolution of geological layers through geological history. The sedimentary layers that constitute the petroleum reservoirs were originally deposited in a flat position within sedimentary basins and they were only deformed later by tectonics. Our method basically consists in first building 3D meshes in the deposition spaces of the various geological units by taking advantage of their flat position and then transposing these meshes in the present space of the model, in which these units are more or less intensely deformed. This step requires to operate an interpolation from the deposition spaces of the model to present day space. In order to make this interpolation efficient, topological models allow to simultaneously handle several different geometric spaces thanks to neighborhood relationship. In our case, the use of the topological model of Generalized Maps [Lie94] allow us to simultaneously describe the situation of the geological layers in their deposition space and in the present-day space. We will show that this approach greatly simplifies the construction of 3D meshes, ensures their better conformation to the layer geometry and allows to keep a fully consistent topology throughout the model building process.

The paper is organized in the following way. We will first present the state of the art regarding 3D meshing for geology and topology description tools. We will then present the workflow that we use and detail its different steps. We will particularly describe the 3D meshing process of stratigraphic units, especially when faults locally disturb the local geology. We will conclude by emphasizing the advantages of the proposed method and by describing how we intend to make it operational in the various geological configurations occurring in subsurface reservoirs.

State of art

Mesh Quality

The meshes resulting from the final stage of the modeling process must be fit to be the support of fluid flow simulations. The simulation quality directly depends on the mesh quality, regarding several requirements.

The meshes must first be oriented everywhere in accordance with the local stratification. Difficulties appear when stratigraphic unconformities are met, since they induce abrupt changes in the layer orientations.

The meshes must also be built in accordance with the fault directions. This requirement adds a difficulty since faults are not always perpendicular to the layer surfaces. The meshes must also be well-formed: homogeneous, mostly composed of cuboids and such that the line joining the centers of two neighbor cells are sub-perpendicular to the edge that separates them. In Figure 3, the left mesh respects the previous constraint and therefore looks better than the right mesh.

Finally, the cell must preferably be oriented perpendicularly to the flow direction that goes from one well to another. This last constraint is more or less contradictory with the two previous ones since once a mesh have been built to satisfy them, it must be deformed to conform with layer and fault orientations. Therefore optimal meshes are difficult to build and necessarily result from compromises.

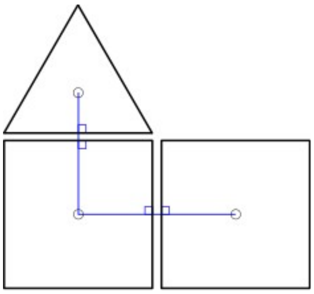

Figure 1 Good quality mesh

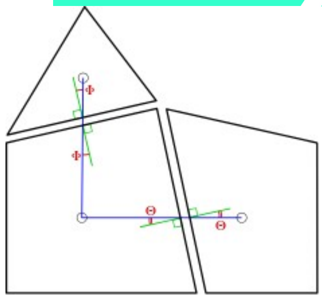

Figure 2 : Bad quality mesh

Figure 3 : Mesh quality representation

Methods and tools

Several methods allow to mesh a 3D geological scene. We can categorize them by the two types of resulting data structures: non-topological structures consisting in structured meshes or unstructured meshes, and complex topological structures [PR13].

Most unstructured meshes are based on tetrahedrons [CSPL+15] and can also be represented as Voronoï meshes. The resulting spatial discretization allows to operated fluid simulations based on finite-element methods but not the widespread ones based on finite-difference methods. Structured meshes essentially use grids, which can sometimes be assimilated to topological structures since they efficiently allow the identification of the cell neighbors. Orthogonal cartesian grids are the simplest grid structures. They are well-adapted to finite-difference or finite-element simulations but they can only handle basic scenes without structural complexity. They have been derived into corner point geometry grids that consist in defining the vertices of the eight corners of each cell. These grids allow to represent more complex and realistic structures but they generate distortions which affect the numerical precision of simulations [Yah13].

Topological approach

Topological data structures separate the topological and the geometric aspects. Topological data structures describe neighbor relationships between cells (vertices, edges, faces, volume) which can be used to efficient process the model. A topological model may also handle various dedicated data -called embeddings- such as geological labels or property values. The geometry is usually one of these embeddings. Since topological data structures allow to create local consistent operations, they ease the definition of meshing operations, by successively modifying the neighbor relationships and the embeddings.

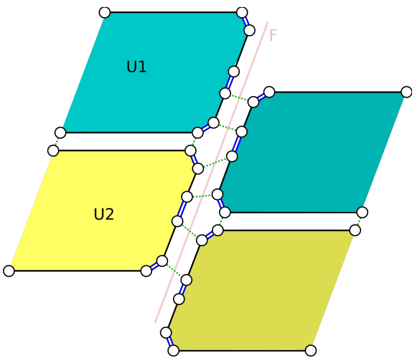

Figure 4: Topological representation of two unities

The topological model proposed in this article lies on the Generalized Maps (G-maps) [Lie94] which explicitly represent the neighbor relations for all cell dimensions. Object consistency is ensured by topological and embedding constraints. The Figure 4 shows a G-map example which exhibits different relationships (colored links) between cells. We can see that along faults, two different blocks may have geometrically common faces. Within a topological model, these faces explicitly share a neighbor relation. These relations prevent undesired holes between mesh cells. They also ease fluid simulation, since they directly determine where the fluid can flow out.

In [Gui06], G-maps are used for representing the 2D parts of a complex geological scenes. All the represented entities are cut along intersections to produce a boundary representation (B-rep) of each unit's block. However the method does not provide a remeshing of these blocks in order to operate simulation.

Topological operations

We use a graph representation of G-maps that allows us to benefit from the graph transformation rules to efficiently prototype modeling operations [BPAF+10, BALG+14]. Mathematically proved, graph transformations are able to graphically define operations while preserving topological and embedding consistency. In practice, using the modeling tool called Jerboa [Jerboa], operations rules are designed with a graphic interface which validates the topological and embedding constraints criteria on the fly. Once correct, rules can then be directly applied within Jerboa to model objects. This approach is therefore particularly well-suited to consistently design the complex operations involved in geomodeling.

Figure 7 presents the example of the Catmull–Clark surface subdivision. This operation defined by the rule of Figure 5 divides any face into multiple quad faces depending of the arity of the original face. Figure 7 shows the respective applications of this operation to the two objects of Figure 3. On the topological aspect, the rule consistently subdivides the face. On the embedding aspect, the rule computes all the geometrical values attached to the new edges and may for instance update their geological identities. Note that this rule is quite simple regarding the complexity of the operated subdivision.

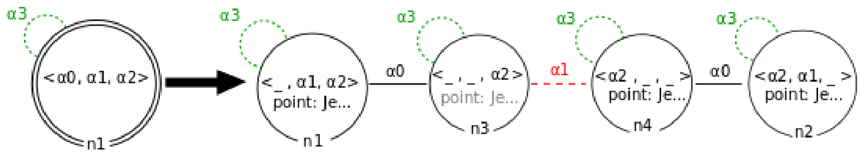

Figure 5 : Graph transformation rule of a subdivision scheme

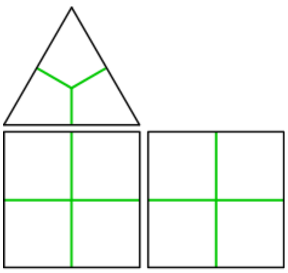

Figure 6 : Good quality mesh subdivided

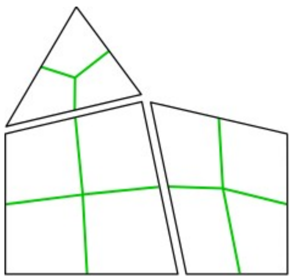

Figure 8: Bad quality mesh subdivided

Figure 7 : Application of the subdivision rule

Proposed workflow

Figure 9 shows the common workflow of our approach. Each step is sequentially detailed in this

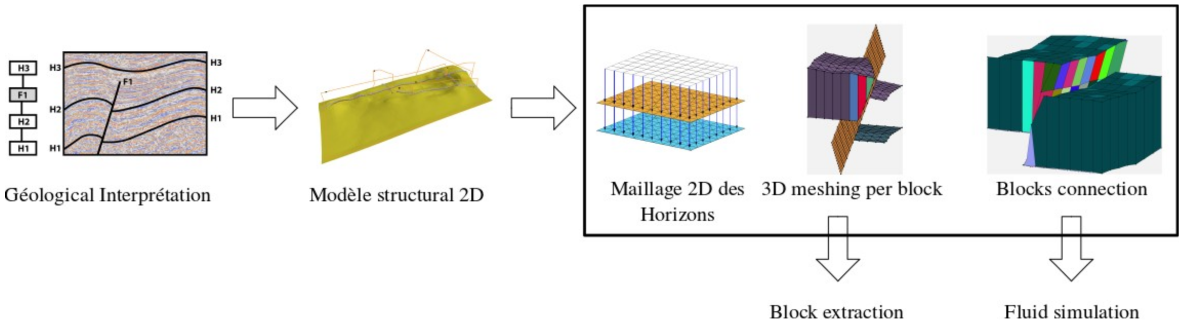

Figure 9 : Workflow

section.

Geological interpretation

The geologists in charge of modeling subsurfaces initially consider raw data issued from both reflection seismic and drilling surveys (cf. geological interpretation in Figure 9). The interpretation process consists in identifying the various geological horizons in view of their signatures (seismic horizons, well markers). This allows the geologists to infer the horizon geometry and by means of a Geological Interpretation Schema (GES) [PR13], they can add geological attributes -age, typology (parallel, unconformable)- to each identified horizon. The geologists also try to identify faults, characterize their geometry and to specify their chronological and topological relationships with the identified horizons. This information concerning faults is itself introduced into the GES in order to constitute a fully formal interpretation of the model.

Geological interpretation is a tedious and under-determined process that deeply depends on geologists' knowledge. Several interpretations and GES's can be attached to one set of raw data. It is therefore essential to keep the memory of all the data and interpretations attached to a given earth model in order to be able to evaluate its reliability and to possibly modify the geological interpretation at any time. This is possible by using the RESQML standard [RESQML] which offers the possibility of encompassing both raw data and geological interpretations.

The horizon and fault interpretations, along their formalized interpretations expressed through GES's, constitute the starting point of the modeling approach that we will now describe. A major advantage of this methodology is its flexibility. It allows the building of several models for one reservoir in accordance with several possible interpretations.

Structural modeling

The building of a structural model constitutes the first important step of the reservoir building process. The structural model consists in an assembly of the geological surfaces, which is consistent both geologically and topologically. It constitutes the "skeleton" of the reservoir model.

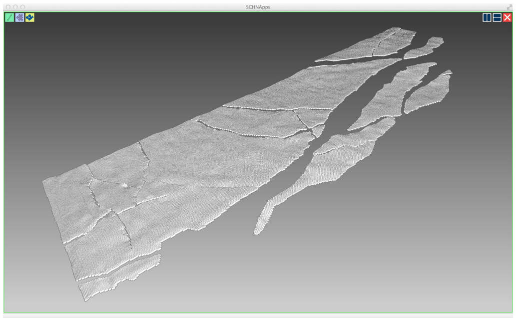

Figure 10 : A raw horizon interpretation

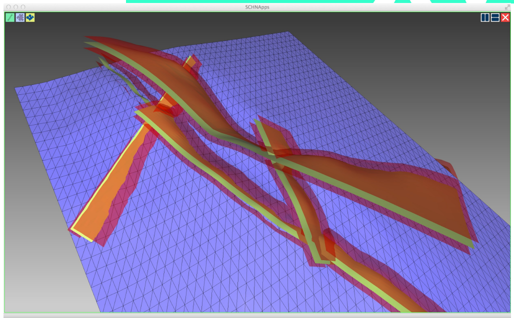

Figure 11 : Surface modeling of the horizon

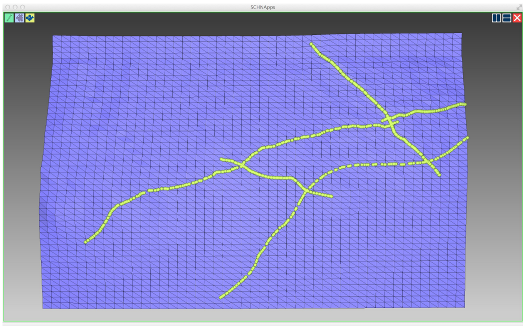

Figure 12 : Computing horizon/fault intersections

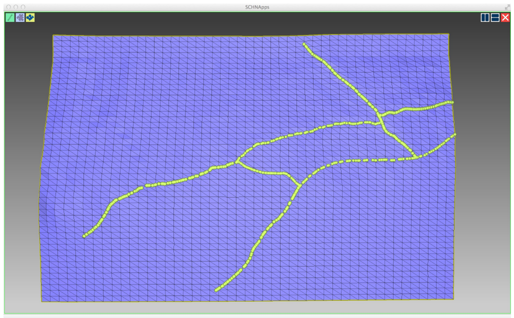

Figure 13 : Processing "STOPS AT" rules

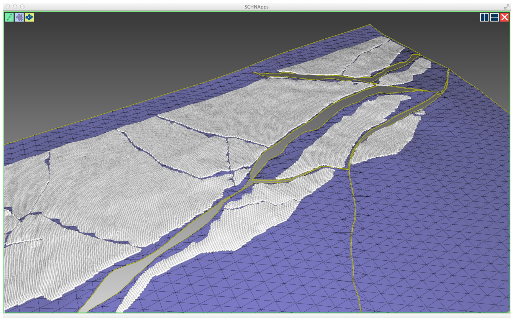

Figure 14 : Opening faults lips

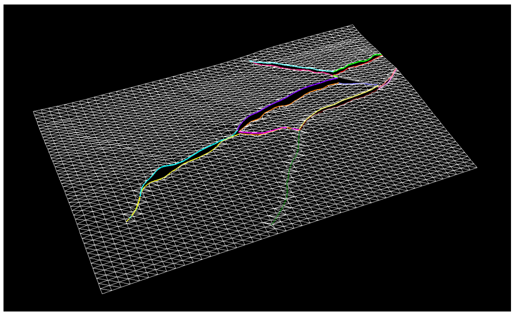

Figure 15 : Identifying horizon/fault contacts

Figure 16 : GeoTopoModeler structural modeling

The Geosiris GeoTopoModeler prototype [GTM] provides tools to specify the various contacts of the structural model (horizon/horizon and horizon/fault contacts) and to accordingly operate the necessary intersections and the removing of the parts of the surfaces that do not enter into the model. The GTM is based on the CGoGN topology-based data structure - which provides an efficient combinatorial maps implementation - in combination with the SCHNApps modeling framework [CGoGN/SCHNApps]. The GTM principles are illustrated in Figure 16. The input of the GTM consists in a RESQML V2.0.1 package containing both raw interpreted data (see Figure 10) and formalized geological knowledge about the subsurface structure. In the first step (see Figure 11), we model the horizon surfaces thanks to a spline analysis of the raw interpreted data [MBA]. For this purpose, we remove unreliable data close to faults. In the next step (see Figure 12), we compute horizon/faults intersections. These intersections are then processed, thanks to semantic knowledge provided by geologists (see Figure 13). For instance, we consider that faults do normally not cross each others but stop one on the others. The last step consists in opening the fault lips according to local geometry (see Figure 14) and in labeling the corresponding horizon/fault contacts. Since each contact is split into a foot-wall and hanging-wall lip, each colored polyline in Figure 15, fits with one labeled contact lip. Our labeling convention allows one:

- to trace back the fault to which a given contact lip is attached,

- to identify the link between the foot-wall and hanging-wall sides of a given contact,
- to identify the link between the contacts lips of the different horizons belonging to a set of stratigraphic units (Stratigraphic Unit Stack [PR13]).

The result provided by the GTM prototype is a 3D surfacic model consisting in a set of horizons surfaces decorated with some labeled contacts.

3D Modeling

In order to build adequate 3D meshes, we need to know the geometry of the various horizons in their initial deposition situation [HBBDR10, PBRB12]. Our topological model defines two geometrical embeddings with their additional semantics mechanism. Initially, the structure is embedded with the geometry obtained from the structural part. It corresponds to what we call the *present geographic space*. By operating unfolding methods [TGM05] on each geological unit, additional geometries are then computed which are those of the *initial deposition space* of each of the processed units. Our topological structure has the ability to switch between the geometries of the geographic and deposition spaces and to ensure topological consistency between each couple of geometries in the course of any modeling operation. Figure 19 shows an example of this double geometry for two horizons. The left part of the figure (see Figure 17) corresponds to their *deposition space* in which the horizons are flat while the right part of the figure (see Figure 18) corresponds to the present geographic space.

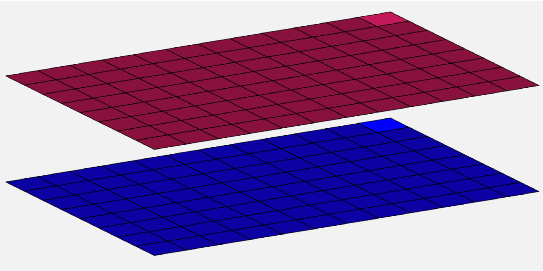

Figure 17 : Horizons in the deposition space

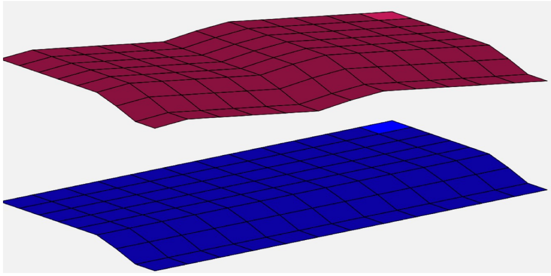

Figure 18 : Horizons in the present geographical space

Figure 19 : Horizons representation in the different spaces

We define a geological block, as a part of a geological unit surrounded by faults. For a 3D meshed model, we first consider the deposition spaces of the various units and, for each one, we project a 2D mesh onto each horizon (top and bottom) of each block. Thereby, the top and bottom meshes are aligned in each deposition space. Moreover, the projection of a single 2D mesh guarantees the concordance between the successive units which are not separated by unconformities.
This feature allows to easily find the link between the top and bottom horizons vertices. In each block, pillars are weaved between the corresponding vertices of the top with the bottom surfaces of the unit in order to define a 3D mesh. Faults are then introduced into this elementary regular 3D mesh and the topology is updated.

A correct topology taking the faults into account is thus established for the meshes attached to each of units in its deposition space. To construct the final meshed model, it then remains to transport these various meshes into the geometric space. The main advantage of our topology based approach is its ability to keep during this operation the topologies constructed in the various deposition spaces. This is the strong point of our method. Details on the 3D mesh modeling within stratigraphic units will now be given hereunder.

Stratigraphic unit 3D meshing

This section provides details about the generation of meshed stratigraphic units fulfilling the constraints imposed by the fluid flow simulation.

Preliminary horizon 2D mesh

The preliminary step consists in limiting the structural mesh within a definite *box of interest* corresponding to the part of the model that the modeler intends to consider. This limitation reduces the complexity of the mesh and guarantees to obtain closed object in our model.

The main step consists in creating quad surfaces in the deposition space from the horizons surfaces. In order to satisfy the concordance constraint, we already mentioned that we use a unique regular grid projected on the top and the bottom horizons of the meshed block (see Figure 20). The regular grid may depend on various parameters such as the cell size or the grid basis (units vectors in its 2D plane). This way of proceeding avoids the possible trouble of mixing the original mesh and our consistent new mesh.

A new surface is produced for each horizon thanks to the regular grid, in which the location of each vertex is led by the original horizon surface. Figure 22 shows the new surfaces composed only of quad faces having all the same Z coordinate. The vertex locations are then interpolated from the deposition space to the geographic space with the nearest face as shown in the Figure 24.

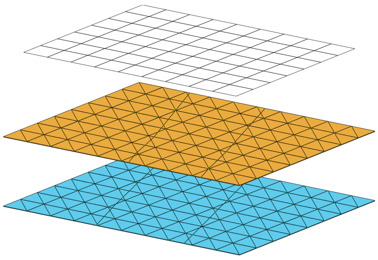

Figure 20 : Triangulated horizons in the deposition space with the regular grid

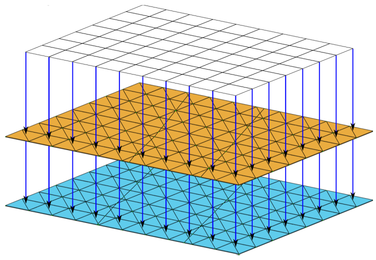

Figure 21 : Projection of the regular grid on horizons

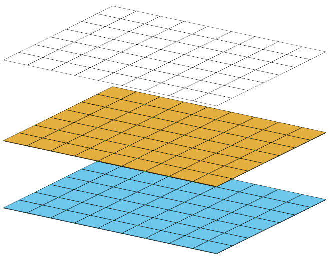

Figure 22 : New horizons surfaces

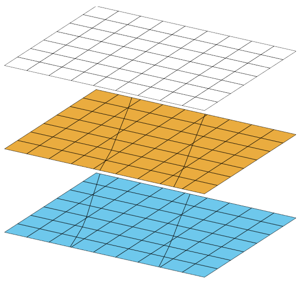

Figure 23 : Fault lips insertion

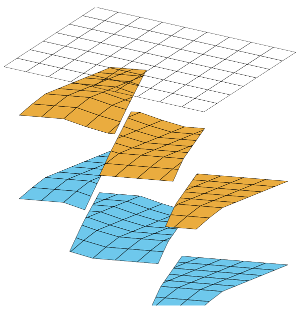

Figure 24 : New horizons surfaces in the present geographical space

Figure 25 : Horizon 2D mesh process

The last step consists in intersecting, each new horizon surface with the original fault surface. A classic Boolean operation [Zho16] is applied based on the fault faces and on the faces of the new horizon surface. The Figure 23 illustrates the complete 2D meshes with fault lips on the embedded regular grids.

Construction of one 3D mesh per block

The condition for modeling the mesh between the top and the bottom surface of a geological unit, is that the two horizons have the same 2D mesh (except for fault lips position). Thus, a quad face can be the top/bottom face of a 3D mesh, and there is no need to apply Boolean operation to the top and the bottom to have two fitting 2D meshes. Thanks to the regular grid projection, this property is already satisfied.

However, some meshes are intersected by faults. To correctly model each part of these meshes in the hanging-wall and foot-wall blocks, it is necessary to determine the position of the fault in the unit deposition space. Of course, the fault did not exist at the time of the unit deposition, but its geographic position corresponds to a defined position in the deposition domain. Due to the sliding of the blocks along the fault, the line of intersection between a horizon and the fault surface has two different positions, one on each side of the fault. These two positions in the geographic space can be computed by interpolation considering the position of the fault surface in the deposition space. And thanks to the richness of our G-map topological model, these various positions of the fault/horizon intersection can be considered as different embeddings of one single topology (see Figure 28). Each vertex of the fault has this three positions: one in the present geographic space, and two in the deposition space. These last positions are computed by interpolation from the positions of fault lips in the deposition space.

3D meshes can thus be created by simply linking the top and bottom of each unit by what we call pillars. The pillar computation is operated in the deposition space. In this space, the pillars are vertical since the same 2D mesh has been used on the top and bottom of the unit, making the pillar construction a simple operation. Nevertheless, this allows the creation of pillars that follow the directions of the layers in the geographic space even if these layers are significantly deformed.

The creation of meshes by means of pillar is simple as long as we remain far from faults. However in the vicinity of a fault, we need to check whether the pillars intersect the faults surface or not. Then, any intersected pillar is considered to have one extremity set on its intersection with the fault surface while the other one remains hooked to an horizon.

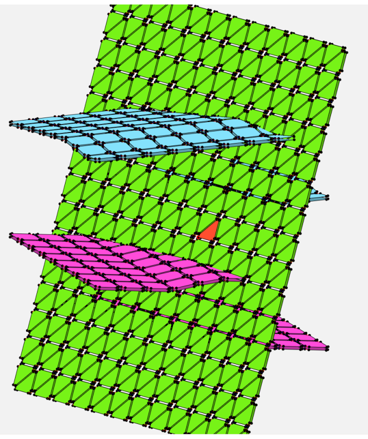

Figure 26 : Example of fault in geographic space

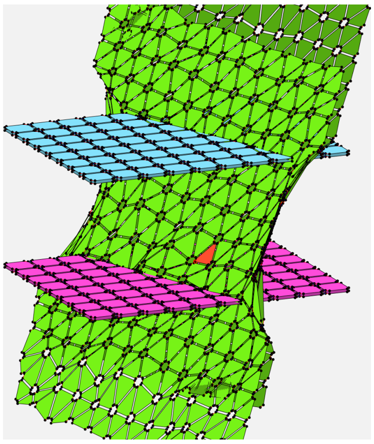

Figure 27 : Fault interpolation in the deposition space

Figure 28 : Representation of the fault position in geographic space and in deposition space

We have obtained consistent meshes far from faults, and open meshes - that have to be closed - near the faults. The next step then consists in creating the missing faces along the fault surface in order to close the blocks. There exist different case of open faces: faces on the side (with just one edge is along the fault), faces in the middle (all edges are along the fault) or faces resulting of multiple cuts by a fault network (see Figure 31).

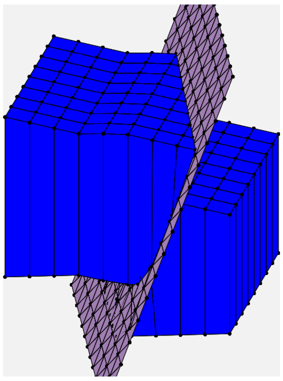

Figure 29 : Blocks with one fault

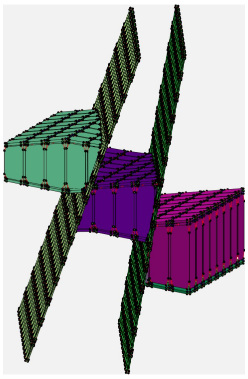

Figure 30 : Blocks with two faults

Figure 31 : 3D mesh per block

Linking blocks

All the blocks can then be connected together. We intend to finalized this part of the process in the coming months. As we mentioned before, linking two blocks belonging to a set of conformable units along an horizon is a simple operation since it just consists in a mesh sewing. But the block linkage is not so easy along faults. In this case, the fault shift induces a lack of concordance across the fault between the meshes belonging to the foot-wall and to the hanging wall.
In this case, to realize the mesh linkage, it is necessary to split each faces of the mesh along the faults surface in order to match it with faces of the opposite blocks, which is a complex Boolean operations. The difficult part is the intersections between the faces. The faces of two blocks along the fault do not belong to one same plane. Therefore, the intersection between the faces must be performed on a common median plane, which must be carefully defined.

Extraction of block boundaries

Boundary representation (B-Rep) is decisive for studying subsurface. Thanks to our topology-based data structure and our modeling process, the geological blocks boundaries can be easily extracted from 3D meshes. This extraction mainly consists in simplifying the blocks meshes.

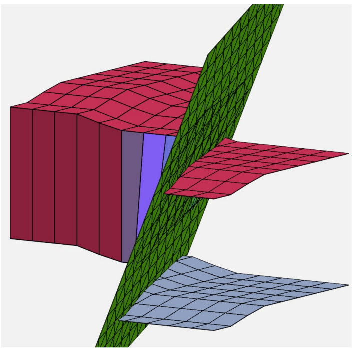

Figure 32 : A single block meshed

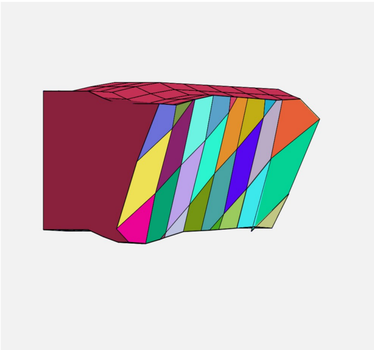

Figure 33 : Simplified block extracted

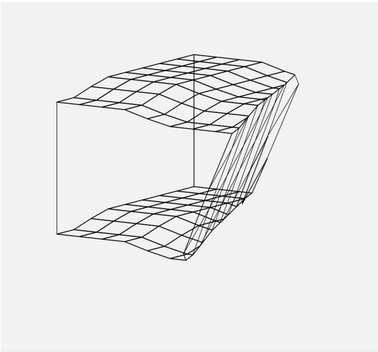

Figure 34 : Extracted block in wireframe

Figure 35 : Block extraction process

We take benefit of the topology to easily determine whether a face is inside a block or on the border. We then remove the inner faces. Then, the unnecessary edges are removed. They are identified by semantic label (side, top, bottom and fault faces of the block) and geometric criteria. This geometric criteria rely on normal vector to characterize coplanar faces.Figure 35 shows different views of one block extraction. Figure 34 illustrates the suppression of inner faces and simplification of the border. Figure 34 shows the simplification of faces which satisfy the geometric criteria, and therefore side faces are collapsed whereas the top and bottom surface remain unaltered. Finally, Figure 33 presents the simplification result from the fault side block.

Conclusion

We have introduced in this paper a 3D subsurface meshing method that is part of a complete geomodeling workflow. Our method relies on a rich topology-based data structure: the G-maps, which allows to efficiently associate several types of data to a given topological structure. These data include two geometries (one in the initial deposition space and the other in the present-day space) and semantic labels for identifying horizons, faults, etc. Moreover, we use graph transformations rules in order to speed up the development of complex modeling operations.
So far, this meshing method already allows us to obtain a 3D subdivision of blocks concordant with horizons and more generally with stratigraphic sets of conformable units. In addition, it allows the extraction of boundary representations of the block outer hulls.

The realized geometric modeling operations have been tested on synthetic data examples (see section 4). We are presently working on real data examples in order to refine and test the scalability of our methodology. In the near future, the method will be extended in order to achieve the fault concordance by replacing the current grid with a suitable 2D mesh. We will also extend our cell division methodology of 3D mesh in order to take into account stratigraphic unconformities (erosions and onlaps). Furthermore, that data exchanges between the various workflow steps will be adjusted in order to be operated through the RESQML format.

References

[BALG+14] Hakim Belhaouari, Agnès Arnould, Pascale Le Gall, Thomas Bellet.
Jerboa: A Graph Transformation Library for Topology-Based Geometric Modeling.
ICGT 2014: 269-284

[BPAF+10] T. Bellet, M. Poudret, A. Arnould, L. Fuchs, P. Le Gall
Designing a topological modeler kernel: a rule-based approach. Proceedings of Shape Modeling International SMI'10, 2010.

[CGoGN/SCHNApps] CGoGN/SCHNApps, http://cgogn.unistra.fr,
Université de Strasbourg, 2016

[CSPL+15] Pauline Collon, Wendy Steckiewicz-Laurent, Jeanne Pellerin, Gautier Laurent, Guillaume Caumon, Guillaume Reichart, Laurent Vaute.
3D geomodelling combining implicit surfaces and Voronoi-based remeshing: A case study in the Lorraine Coal Basin (France). Computers & Geosciences 77: 29-43 (2015)

[GTM] GTM - GeoTopoModeler, http://www.geosiris.com/geotopomodeler/, Geosiris, 2016

[Gui06] Nicolas Guiard.
Construction de modèles géologiques 3D par co-raffinement de surfaces.
PhD of Ecole des Mines de Paris. May 2006.

[HBBDR10] Horna, S., Bennis, C., Borouchaki, H., Delage, C., Rainaud, J.-F.
Isometric Unfolding of Stratigraphic Grid Units for Accurate Property Populating: Mathematical Concepts.
EAGE. European Conference on the Mathematics of Oil Recovery (ECMOR), september 2010,
Oxford, United Kingdom.

[IMMA10] Islam, M. R., Mousavizadegan, S.H., Mustafiz, S. and Abou-Kassem, J. H.
(2010) Index, in Advanced Petroleum Reservoir Simulations,
John Wiley & Sons, Inc., Hoboken, NJ, USA. doi: 10.1002/9780470650684.index

[Jerboa] http://xlim-sic.labo.univ-poitiers.fr/logiciels/Jerboa/?lang=en, Université de Poitiers, 2014.

[Lie94] P. Lienhardt.
N-dimensional generalised combinatorial maps and cellular quasi-manifolds.
International Journal of Computational Geometry and Applications, 1994.

[MBA] MBA - Multilevel B-Spline Approximation Library, https://www.sintef.no/projectweb/geometry-toolkits/mba/
SINTEF, november 2013

[PBRB12] Poudret, M., Bennis, C., Rainaud, J.-F., Borouchaki, H.
A volume flattening methodology for geostatistical petrophysical properties estimation. Springer.
Proceedings of the 20th International Meshing Roundtable, 569--585, 2012.

[PR13] Michel Perrin, Jean-François Rainaud.
Shared Earth Modeling - Knowledge driven solutions for building and managing subsurface 3D geological models.
TECHNIP. Feb 2013.

[RESQML] Energistics. 2009c. RESQML Standards, http://energistics.org/resqml-standards (accessed 2009).

[TGM05] Boris Thibert, Jean-Pierre Gratier, Jean-Marie Morvan.
A direct method for modeling and unfolding developable surfaces and its application to the Ventura Basin (California). Journal of Structural Geology 27(2). Feb. 2005

[Yah13] Brahim YAHIAOUI.
Maillage dynamique tridimensionnel pour la simulation de l'écoulement dans un bassin sédimentaire.
PhD UTT. 2013.

[Zho16] Zhou, Q., Grinspun , E., Zorin D., Alec Jacobson
Mesh Arrangements for Solid Geometry
ACM SIGGRAPH 2016